ECE-201 : Notes

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Note: Section numbers do not correspond to zyBooks section numbers.

Contents

1 Quick Reference

1.1 Derivatives

$$\begin{aligned}
f(x) = n, &\quad f'(x) = 0 \\
f(x) = x^n, &\quad f'(x) = nx^{n-1} \\
f(x) = g(x)h(x), &\quad f'(x) = g(x)h'(x) + g'(x)h(x) \\
f(x) = \frac{g(x)}{h(x)}, &\quad f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{h(x)h(x)} \\
f(x) = e^{nx}, &\quad f'(x) = ne^{nx} \\
f(x) = \ln(x), &\quad f'(x) = \frac{1}{x} \\
f(x) = sin(nx), &\quad f'(x) = ncos(nx) \\
f(x) = cos(nx), &\quad f'(x) = -nsin(nx)
\end{aligned}$$

1.2 Integrals

$$\begin{aligned}
f(x) = x^n, &\quad \int f(x)dx = \frac{x^{n+1}}{n} \\
f(x) = e^{nx}, &\quad \int f(x)dx = \frac{e^{nx}}{n} \\
f(x) = cos(nx), &\quad \int f(x)dx = \frac{sin(nx)}{n} \\
f(x) = sin(nx), &\quad \int f(x)dx = \frac{-cos(nx)}{n}
\end{aligned}$$

1.3 Natural Log

$$\begin{aligned}
e^{-\infty} &= 0 \\
e^0 &= 1 \\
e^1 &= 2.718 \\
e^\infty &= \infty \\
\ln(e^x) &= x \\
\ln(1) &= 0 \\
\ln(x * y) &= \ln(x) + \ln(y) \\
\ln(x/y) &= \ln(x) - \ln(y) \\
\ln(x^y) &= y * \ln(x)
\end{aligned}$$

1.4 Constants

$\epsilon_0 = 8.85 * 10^{-12}$	Permittivity
$\mu_0 = 4\pi * 10^{-7}$	Magnetic Permability

1.5 Equations

$V = IR$	Ohm's Law
$i = \frac{dq}{dt}$	Current
$v_{ab} = \frac{dw}{dq}$	Voltage
$p = \frac{dw}{dt}$	Power
$p = vi$	Power
$0 = \sum_{k=1}^{n} p_k$	Law of Conservation of Power
$W = VIT$	Total energy where T is time in hours

$S = \frac{1}{\Omega}$	Conductivity
$\Omega = \frac{1}{\sigma}$	Resistivity
$R = \frac{l}{A\sigma}$	Resistance of a wire
$R = \frac{l\rho}{A}$	Resistance of a wire
$R = R_0(1 + \alpha T)$	Resistance varied with temperature
$R = R_1 + R_2 + ... + R_n$	Equivalent Resistance in Series
$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$	Equivalent Resistance in Parallel
$i_i = i_s(\frac{R_{eq}}{R_i})$	Current across a resistor in parallel
$v_i = v_s(\frac{R_i}{R_{eq}})$	Voltage across a resistor in series
$0 = \sum_{n=1}^{N} i_n$	Kirchoff's Current Law
$0 = \sum_{n=1}^{N} v_n$	Kirchoff's Voltage Law

$\epsilon_r = \frac{\epsilon}{\epsilon_0}$	Relative Permittivity
$\epsilon = \epsilon_0(1 + \chi_e)$	Permittivity and Susceptibility relation
$E = \frac{q}{\epsilon A}$	Electric field strength of a parallel plate capacitor
$E = \frac{v}{d}$	Electric field strength of a parallel plate capacitor
$C = \frac{q}{v}$	Capacitance
$C = \frac{\epsilon A}{d}$	Capacitance (parallel-plate)
$C = \frac{2\pi\epsilon\ell}{\ln(b/a)}$	Capacitance (coaxial)
$i = C\frac{dv}{dt}$	Current in a capacitor
$p(t) = Cv\frac{dv}{dt}$	Instantanous power of a capacitor
$w(t) = \frac{1}{2}Cv^2$	Instantanous energy of a capacitor
$\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}$	Equivalent Capacitance in Series
$C_{eq} = \sum_{i=1}^{N} C_i$	Equivalent Capacitance in Parallel

$L = \frac{\mu N^2 S}{\ell}$	Inductance of a solenoid
$\Lambda = i\frac{\mu N^2 S}{\ell}$	Magnetic Flux linkage of a solenoid
$L = \frac{\Lambda}{i}$	Inductance
$\mu_r = \frac{\mu}{\mu_0}$	Relative Magnetic Permeability
$v = \frac{d\Lambda}{dt}$	Faraday's Law
$v = L\frac{di}{dt}$	Faraday's Law
$i(t) = i(t_0) + \frac{1}{2}\int_{t_0}^{t} v d\tau$	Current of an Inductor
$p(t) = Li\frac{di}{dt}$	Power of an Inductor
$w(t) = \frac{1}{2}Li^2$	Energy of an Inductor

$\tau = R_{eq} * C$	Time constant for RC circuits

1.6 Tables

1.6.1 SI Units

Dimension	Unit	Abbrev
Charge	coulomb	C
Voltage	volt	V
Resistance	ohm	Ω
Capacitance	farad	F
Inductance	henry	H
Frequency	hertz	Hz
Energy	joule	J
Power	watt	W
Current	ampere	A

1.6.2 Metric Prefixes

Prefix	Symbol	Magnitude		Prefix	Symbol	Magnitude
exa	E	10^{18}		atto	a	10^{-18}
peta	P	10^{15}		femto	f	10^{-15}
tera	T	10^{12}		pico	p	10^{-12}
giga	G	10^{9}		nano	n	10^{-9}
mega	M	10^{6}		micro	μ	10^{-6}
kilo	k	10^{3}		milli	m	10^{-3}

1.6.3 American Wire Gauge (AWG) System

Gauge	Diameter (mm)
0	8.3
2	6.5
4	5.2
6	4.1
10	2.6
14	1.6
18	1.0
20	0.8

1.6.4 Resistor Color Codes

Color	Digit	Multiplier	Tolerance
Silver		10^{-2}	10 %
Gold		10^{-1}	5 %
Black	0	10^0	
Brown	1	10^1	1 %
Red	2	10^2	2 %
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	0.5 %
Blue	6	10^6	0.25 %
Purple	7	10^7	0.1 %
Gray	8		
White	9		

1.6.5 Relative Electrical Permittivity of Common Insulators

Material	Relative Permittivity ϵ_r
Air (at sea level)	1.00006
Teflon	2.1
Polystyrene	2.6
Paper	2-4
Glass	4.5-10
Quartz	3.8-5
Bakelite	5
Mica	5.4-6
Porcelain	5.7

1.6.6 Relative Magnetic Permeability of Material

Material	Relative Permeability μ_r
All Dielectrics and Non-Ferromagnetic Metals	1
Cobalt	250
Nickel	600
Mild Steel	2000
Iron (pure)	4000-5000
Silicon Iron	7000
Mumetal	100000
Purified Iron	20000

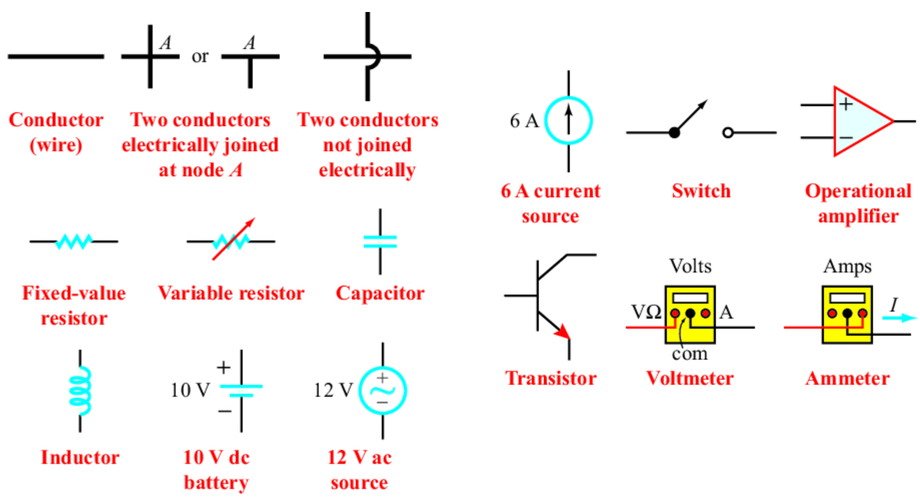
2 Introduction to Circuits

2.1 Circuit Representation

2.1.1 Terminology

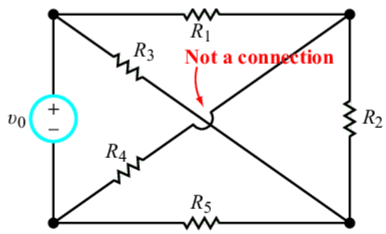
Node : An electrical connection between two or more elements.
Ordinary Node : An electrical connection that only connects two elements
Extraordinary Node : A node connected to three or more elements
Branch : Trace between two consecutive notes with only one elements between them
Path : Continuous sequence of branches with no node encountered more than once
Extraordinary Path : Path between two adjacent extraordinary nodes
Loop : Closed path with the same start and end node
Independent Loop : Loop containing one or more branches not contained in any other independent loop
Mesh : Loop that encloses no other loops
In Series : Elements that share the same current. They have only ordinary nodes between them. Series elements connect only at one end. If the elements share both ends, they are in parallel.
In Parallel : Elements that share the same voltage. They share two extraordinary nodes.

2.1.2 Symbols

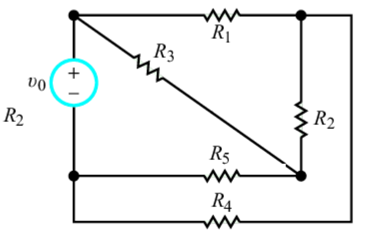


2.1.3 Planar Circuits

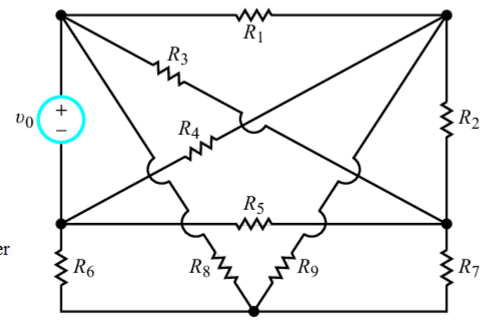
A circuit is planar if you can draw it in a 2D plane without any branches crossing over each other. If this can't be done, then the circuit is nonplanar. See example below.



(a) Original circuit



(b) Circuit redrawn to eliminate crossover



(c) Nonplanar circuit

2.2 Charge and Current

2.2.1 Charge

Charge has the following properties:

- Charge can be positive or negative
- The smallest quantity of charge is that of a single electron or proton. It's magnitude is usually denoted by e
- According to the law of conservation of charge, the net charge in a closed region can not be created nor destroyed
- Opposites attract, likes repel

Unit for charge is coulomb (C) (see 1.1) and the magnitude of $e = 1.6 * 10^{-19}$

Two symbols for charge (q)

- Proton: $q_p = e$
- Electron: $q_e = -e$

Since net charge, also just called charge, is always equal to $q_p + q_e$, charge is always a multiple of e

2.2.2 Current

First, don't forget Ohm's Law. $I = V/R$

Current is caused by electron drift. That's when electrons transfer from one atom to another. A single electron is not moving all the way down the wire. One atom gives his neighbor an electron, which then gives another electron over.

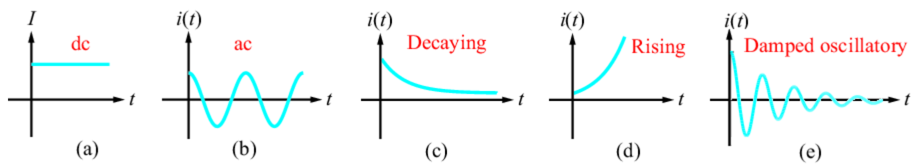
Electrons move in the opposite direction of current. Current flows positive to negative. Electrons move negative to positive.

The amount of current flowing through a wire is equal to the amount of charge that crosses the wire over a period of time.

$$i = \frac{dq}{dt}$$

There are different types of current too. There's DC (direct current) which is when the electrons move one way the whole time. AC (alternating current) is when the electrons move back and forth and wiggle. Then from there, DC can be decaying or rising, and damped AC oscillations.

For notations, uppercase letters (V, I) are used for DC quantities with no time variation which lowercase (v, i) are used in general cases.



2.3 Voltage and Power

Current and voltage are the big guys in circuit analysis (duh ohms law baby). Current refers to the movement of electric charge while voltage is the concentration of that charge.

$$V_{ab} = \frac{dw}{dm}$$

We call V_{ab} the gravitational voltage when dw is the potential energy change and dm is the mass between heights a and b. Remember that potential energy is the amount of energy something higher of the ground has (dummy definition).

Electrical voltage deals with polarity and the attraction of opposite charges.

$$v_{ab} = \frac{dw}{dq}$$

Here, v_{ab} is the electrical voltage between two points, either in space or on a wire, where dw is the energy (joules) required to move positive charge dq from b to a. This is the same for moving negative charge from a to b.

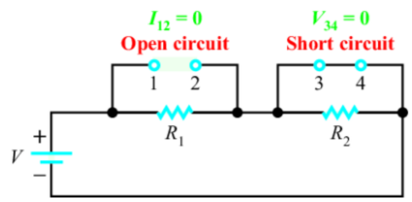
Just as before with potential energy, the voltage is a potential difference. If v_{ab} is positive, point a is at a higher potential than b. We would say there is a "voltage rise" from b to a or a "voltage drop" from a to b.

Since voltage is a potential difference, the place with the lowest potential is ground. Ground is where the 0V reference point is.

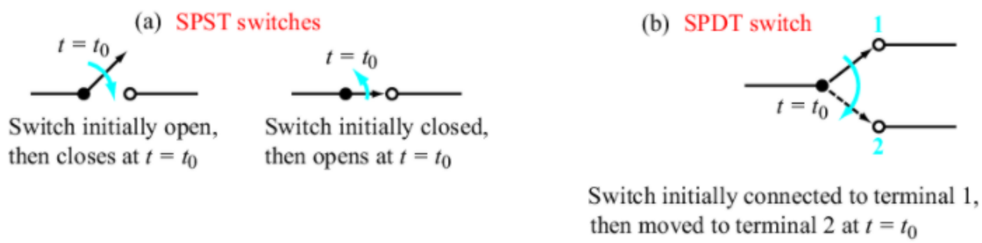
When measuring voltage with a voltmeter, you test in parallel with what you're checking. This is because the voltage drop across branches is the same. When testing current with an ammeter, you test in series since current is the same in a single branch. In the case of a voltmeter, it uses a minimal amount of current so as not to affect the circuit, while an ammeter uses a minimal amount of voltage.

2.3.1 In Circuits

Open Circuit : A discontinuity in a circuit. Infinite resistance. Zero current. See image below.
Short Circuit : Complete continuity in a circuit. Zero resistance. No voltage drop. See image below.



Switch : Controls current flow. When open/off, acts as an open circuit. When closed/on, acts as a short circuit.
SPST Switch : Single-pole single-throw switch
SPDT Switch : Single-pole double-throw switch
Terminal : Where the switch connects to things in the circuit



2.3.2 This Time Just Power

A power supply (battery) creates a voltage rise, while a power recipient (light bulb) creates a voltage drop.

Power is measured in watts and can be expressed using $p = vi$ but how do we get that. First, remember power p is the change in energy dw over the change in time dt.

$$p = \frac{dw}{dt}$$

Given the v_{ab} and i equations, we can derive the following:

$$p = \frac{dw}{dt}$$
$$p = \frac{dw}{dq} * \frac{dq}{dt}$$
$$p = vi$$

The law of conservation of power means that the sum of all power is a circuit will be zero. So a circuit of n elements:

$$\sum_{k=1}^{n} p_k = 0$$

Power supplies sometimes have ratings assigned that describe their abilities to deliver energy. A 9V battery may have an output capacity of 200 ampere-hours (Ah) which means it can deliver a certain current I over a time T at 9 volts such that $IT = 200$.

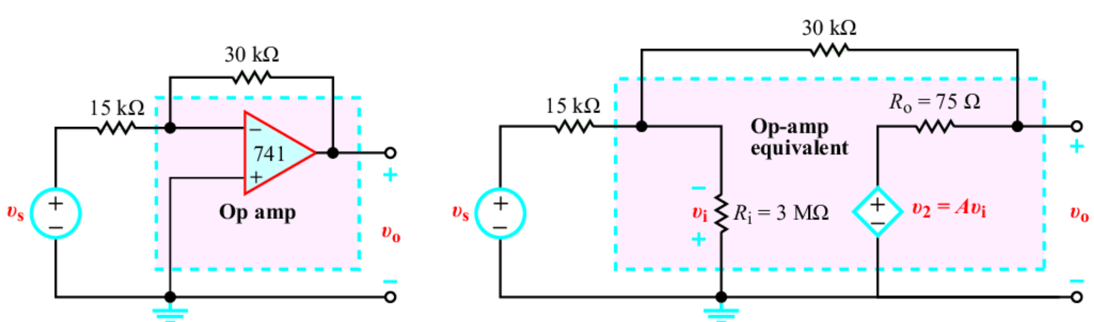
The total energy a power supply can supply can be given by $W = VIT$ where T is time in hours. Total energy can also be calculated with $W = VQ$ since $v(t) = dw/dq$.

2.4 Building a Circuit

All circuits can be represented in terms of a different, equivalent circuit composed of basic elements with idealized characteristics. The set of basic elements commonly used includes:

- Voltage and current sources
- Passive elements
  - Resistors
  - Capacitors
  - Inductors
- Switches

Below is an example of an equivalent circuit where the complex operational amplifier element was replaced with a circuit comprised of basic element like the ones listed above.



(a) Op-amp circuit (b) Equivalent circuit with dependent voltage source

As an extension to Ohm's law, we can see the i-v relationship for a resistor as written below. This is a linear relationship so long as R is constant. A circuit with only elements with i-v relationships (a circuit with constant resistance) is a linear circuit.

$$i = \frac{v}{R}$$

2.4.1 Independent and Dependent Sources

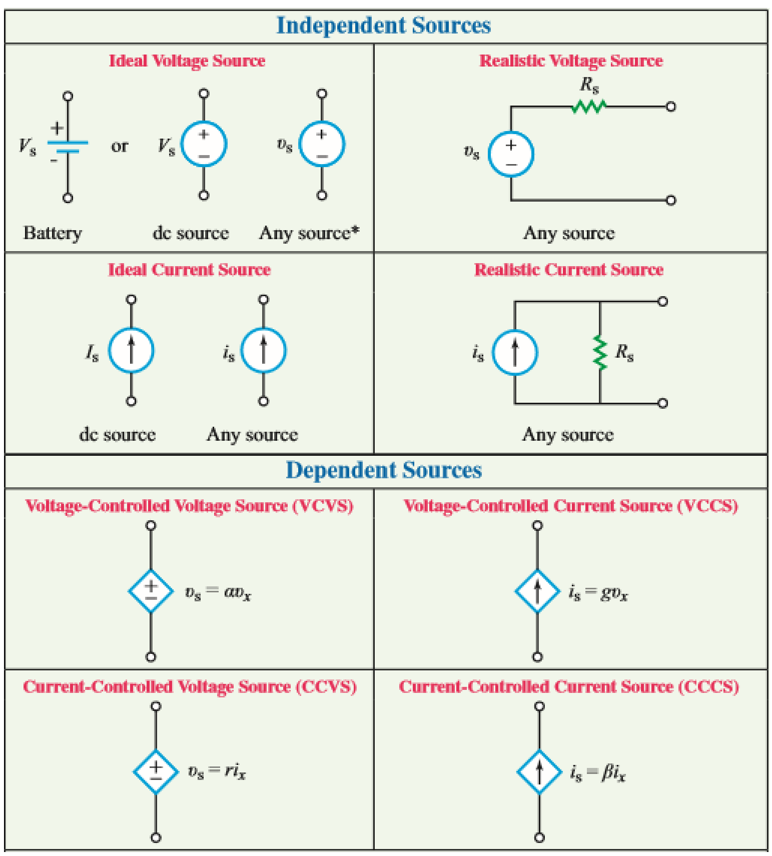
An independent voltage source is any voltage source that is not dependent on current and has the following i-v relationship. An independent voltage source is anything that generates voltage. It could be a generator or a battery.

$$v = V_s \text{ for any } i \neq \infty$$

An independent current source is any current source that is not dependent on voltage. In practice, there is no such thing as a "current battery" that keeps current flowing in a circuit. We can build that circuit and we can use that in describing circuits. An ideal independent current source has the following i-v relationship.

$$i = I_s \text{ for any } v_s \neq \infty$$

A dependent voltage source is a voltage source that relies on either current or voltage to output a certain voltage. In the op-amp image above, you can see that there is a VCVS, or a Voltage Controlled Voltage Source. Since the op-amp is very complex containing transistors and capacitors and diodes, it produces a unique behavior that can simply described with the VCVS. There also exists CCVS which are Current Controlled Voltage Sources.



3 Laws and Anaylsis

3.1 Ohm's Law

Resistive Circuit : A circuit with only sources and resistors

Conductivity : The measure of how easily electrons can drift through material when voltage is applied. σ. Measured in Siemens per meter(S/m). A siemen is the inverse of an Ohm. $S = 1/\Omega$

Resistivity : The inverse of conductivity $\rho = 1/\sigma$

Conductor : Conducts electricity

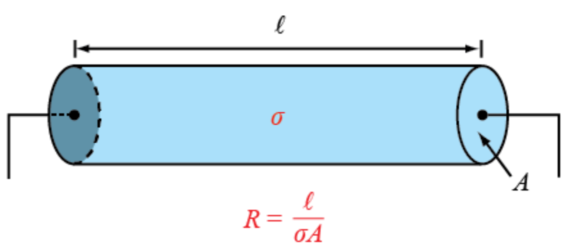
Semi-Conductor : Conducts some electricity

Dielectrics : Does not conduct electricity. An insulator. "Die - electric"

3.1.1 Resistance

Two main factors in the resistance of a device

- The inherent bulk property of its material to conduct current
- The shape and size of the device



Given the above longitudinal resistor, it's resistance is given below where length is l, conductivity is σ, resistance is ρ, and the cross sectional area is A.

$$R = \frac{l}{\sigma A} = \rho\frac{l}{A}(\Omega)$$

Thermistor : R sensitive to temperature

Piezoresistor : R sensitive to pressure

Light Dependent Resistor (LDR) : R sensitive to light intensity

Rheostat : 2 terminal variable resistor

Potentiometer : 3 terminal variable resistor

Resistance can vary by temperature as given below where R is the final resistance, T is the temperature in Celsius, R_0 is resistance at $T = 0°C$ and α is the materials temperature coefficient.

$$R = R_0(1 + \alpha T)$$

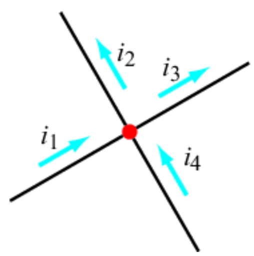
The inverse of resistance is conductance as seen below. Conductance is measured in siemens (S), also called mho. It also can be expressed as Ω^{-1}

$$G = \frac{1}{R}(S)$$

3.2 Kirchoff's Current Law (KCL)

The sum of currents entering a node must always equal 0. Remember, current is negative when flowing from negative to positive and positive when flowing positive to negative.

$$\sum_{n=1}^{N} i_n = 0$$

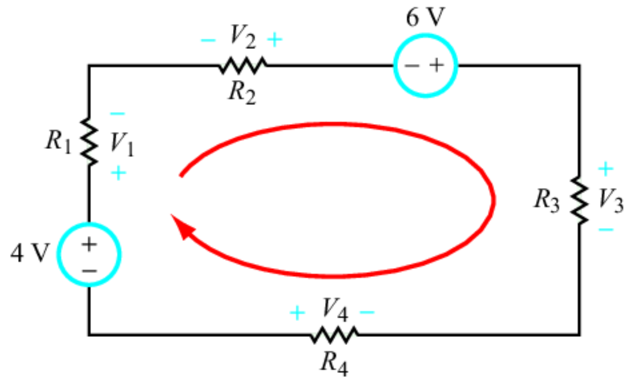


So in the above image, we would express this mathematically as, $i_1 + (-i_2) + (-i_3) + i_4 = 0$, we could also rewrite this as $i_1 + i_4 = i_2 + i_3$ since i_1 and i_4 are entering the node and i_2 and i_3 are leaving the node.

3.3 Kirchoff's Voltage Law (KVL)

The sum of voltages in a loop must always equal 0. Remember, voltage is negative when flowing from positive to negative and positive when flowing negative to positive.

$$\sum_{n=1}^{N} v_n = 0$$



So in the above image, we would express this mathematically as, $4 + (-V_1) + V_2 + 6 + (-V_3) + V_4 = 0$. We can also write this as the sum of the voltage drops is equal to the sum of the voltage rises which would be $4 + V_2 + 6 + V_4 = V_1 + V_3$.

3.4 Equivalent Circuits

We can use equivalent circuits to further simplify circuits to make analysis easier. This involves breaking a circuit down into smaller equivalent circuits to make life easier.

Resistors connected in series cab be simplified using KVL.

$$
\begin{aligned}
0 &= -v_s + R_1 i_s + R_2 i_s + R_3 i_s + R_4 i_s + R_5 i_s \\
v_s &= R_1 i_s + R_2 i_s + R_3 i_s + R_4 i_s + R_5 i_s
\end{aligned}
$$

This is very complicated so we can, using hte distributive property, simplify this.

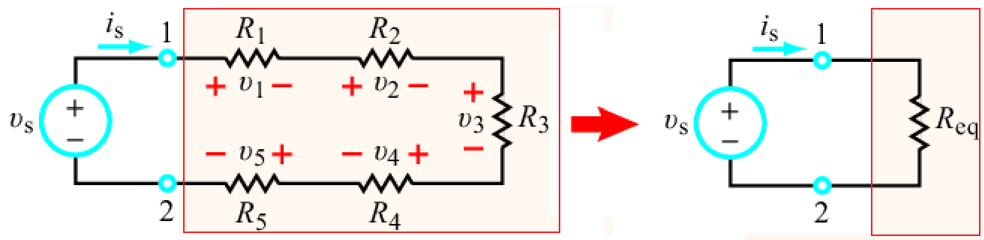
$$
\begin{aligned}
R_{eq} i_s &= R_1 i_s + R_2 i_s + R_3 i_s + R_4 i_s + R_5 i_s \\
R_{eq} i_s &= i_s(R_1 + R_2 + R_3 + R_4 + R_5) \\
R_{eq} &= R_1 + R_2 + R_3 + R_4 + R_5
\end{aligned}
$$

The voltage for each of these resistors would be given by

$$v_i = (\frac{R_i}{R_e q}) v_s$$

In the image below, we can see the equivalent circuit.

Combining In-Series Resistors



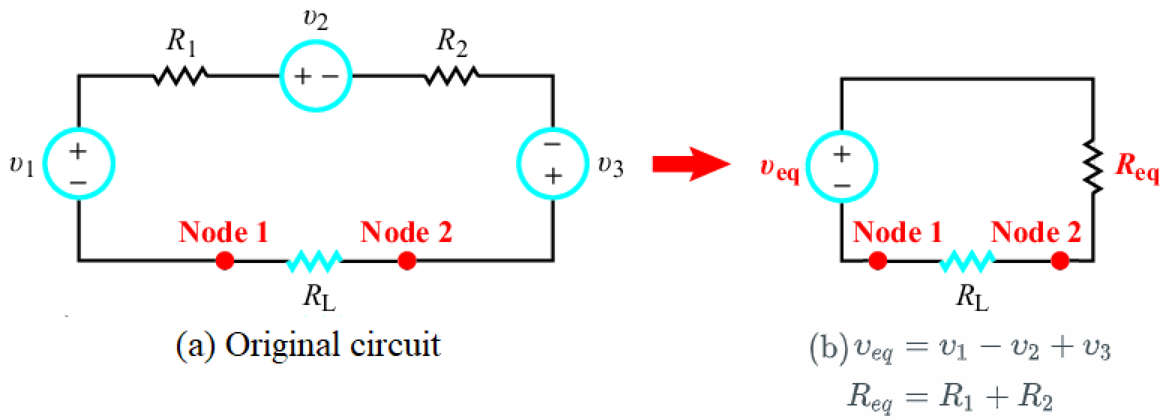
(a) Original circuit (b) Equivalent circuit

In series, we cannot have current sources with different magnitude or directions. This will break the circuit. However, in series, voltage sources can be added, just the way resistors are.

$$\sum_{i=1}^{N} v_i = v_{eq}$$

So in the circuit below, we can say the following:

$$v_{eq} = v_1 - v_2 + v_3$$
$$R_{eq} = R_1 + R_2$$



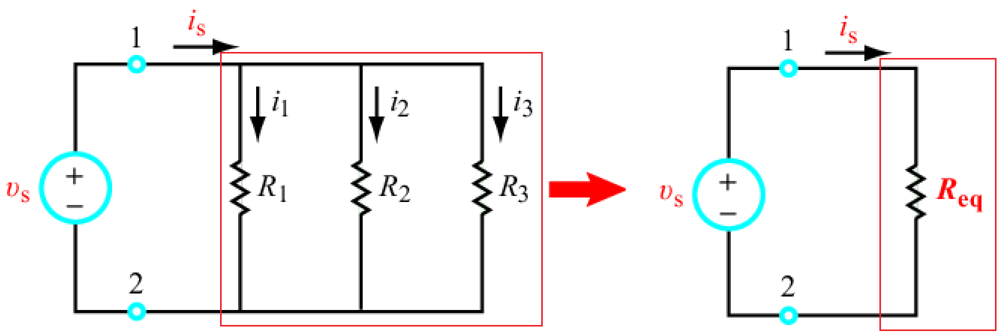
Now when dealing with equivalent circuits in parallel, we can once again derive equivalent resistance, but this time from KCL.

$$i_s = \frac{v_s}{R_{eq}}$$
$$\frac{v_s}{R_{eq}} = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3}$$
$$\frac{v_s}{R_{eq}} = v_s(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3})$$
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R_{eq}} = \sum_i = 1^N \frac{1}{R_i}$$

This means, the current flowing through a resistor in parallel is:

$$i_i = i_s(\frac{R_{eq}}{R_i})$$

Now we see this in the circuit below:



(a) Original circuit (b) $R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$

We can also use the above to show that a realistic voltage source composed of an ideal voltage source in series with a resistor can be replaced with a realistic current course composed of an ideal current source in parallel with a resistor.

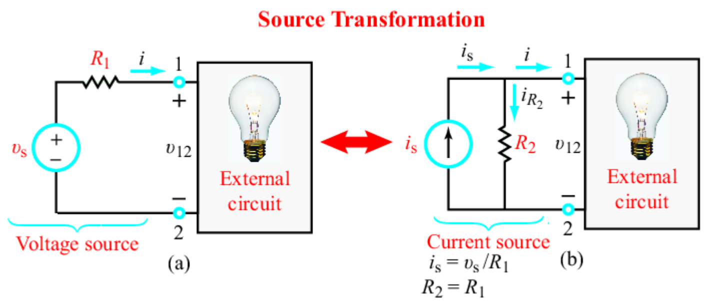
In the circuit below, using KVL, we can see that

$$-v_s + iR_1 + v_{12} = 0$$

which we can transform into

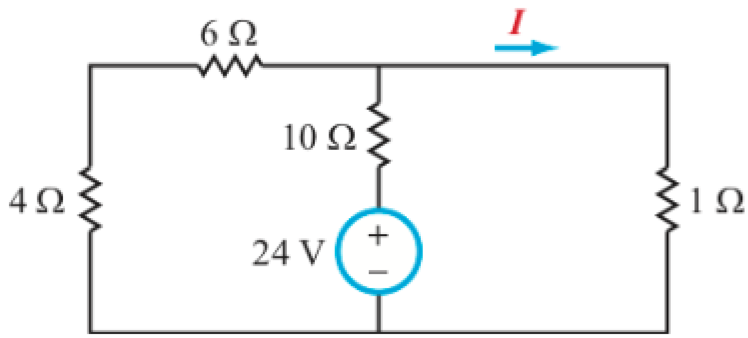
$$i = \frac{v_s}{R_1} - \frac{v_{12}}{R_1}$$

We can see this equivalence in the circuits below.



3.5 Node Voltage Analysis

A way of analyzing current and voltages in a circuit. Take the circuit below.



- Identify the extraordinary nodes and label them
  - There are two here
- Mark one as ground and 0V
  - The top will be V_A the bottom will be ground
- Calculate the voltage in each branch and add the sum to 0
  - See the equation below for the above circuit, each branch is calculated, then added left to right

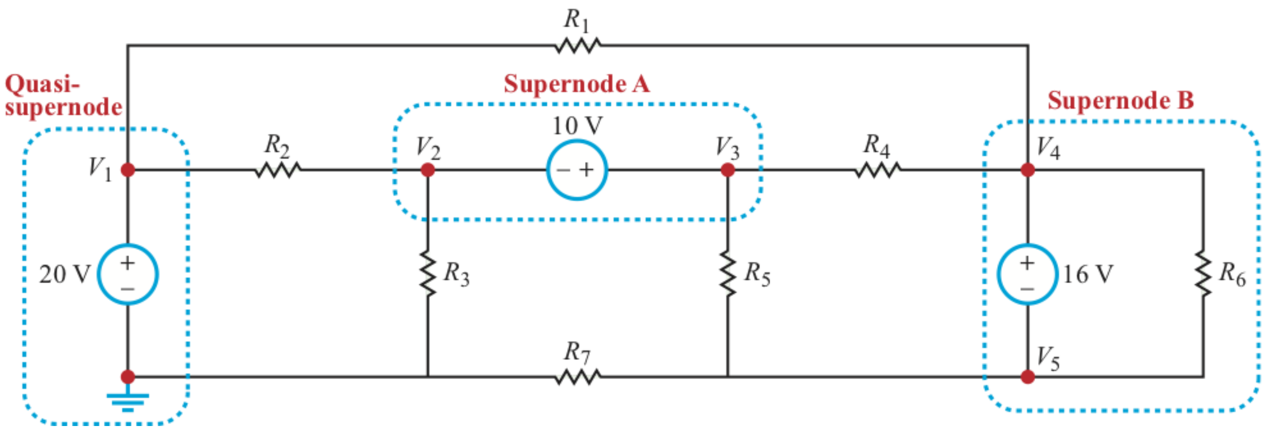
$$\frac{V_A}{6+4} + \frac{V_A - 24}{10} + \frac{V_A}{1} = 0$$

- Solve for V_A and I
  - $V_A = 2V, I = 2A$

3.6 Supernodes

Supernode : The combination of two extraordinary nodes between which a voltage source exists
Quasi-Supernode : A supernode where one of the nodes is a ground node

Supernodes can contain a parallel branch with a resistor on it.



When analyzing circuits, a supernode can be treated as a single node. In the 20V node above we see the following.

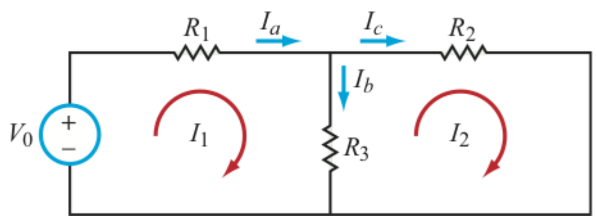
$$
\begin{aligned}
0 &= I_1 + I_2 + I_3 && \text{KCL for } V_2 \\
0 &= I_4 + I_4 + I_6 && \text{KCL for } V_3 \\
0 &= I_1 + I_2 + I_5 + I_6 && \text{Supernode equation since} I_3 = -I_4 \\
10 &= V_3 - V_2
\end{aligned}
$$

Then, in further node analysis, we'd treat V_3 and V_2 as the same node

3.7 Mesh Current Analysis

As an extension of KVL, we can apply the Mesh-Current analysis to find current.

In the below, we already know from KCL that $I_a = I_b + I_c$. When adding mesh analysis to this, we also see that $I_a = I_1$ and $I_c = I_2$. This means $I_b = I_1 - I_2$ since the I_1 current flows through R_3 with I_b and I_2 flows opposit to I_b.



1. Identify all meshes and assign each and unknown mesh current. Define all mesh currents to be clockwise. In the above picture, these are I_1 and I_2
2. Apply KVL to each mesh
3. solve the system of equations

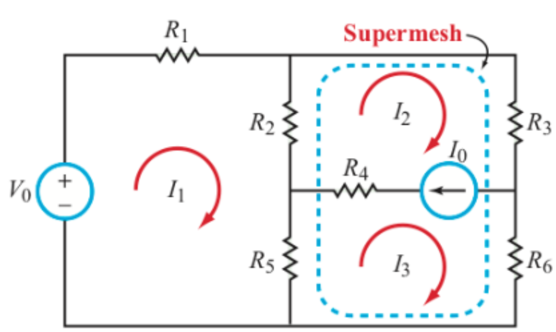
If we apply KVL to mesh 1 and mesh 2, we get the following:

$$\begin{aligned} 0 &= -V_0 + I_1R_1 + (I_1 - I_2)R_3 && \text{Mesh 1} \\ 0 &= I_2R_2 + (I_1 - I_2)R_3 && \text{Mesh 2} \end{aligned}$$

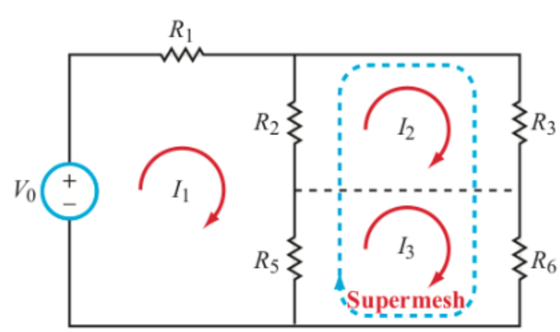
Now simplify and solve for the variables.

3.7.1 Supermeshes

A super mesh is when two meshes share a branch with a current source. This branch can be “removed” and treated a mesh itself.



(a) Two adjoining meshes sharing a current source constitute a supermesh.

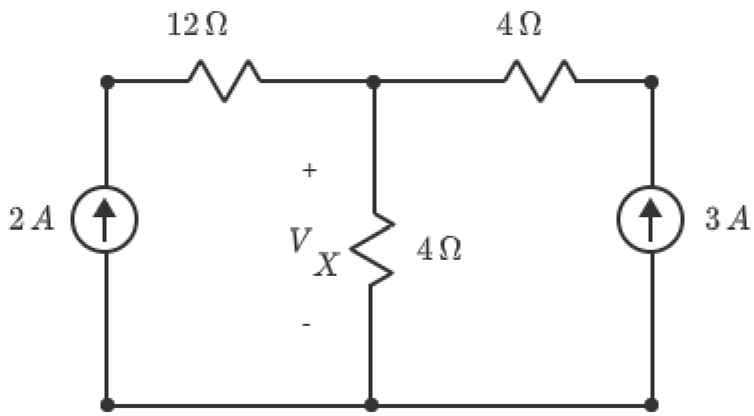


(b) Meshes 2 and 3 can be combined into a single supermesh equation, plus an auxiliary equation $I_0 = I_2 - I_3$.

In the above image, the supermesh is I_0 and be described as $I_0 = I_2 - I_3$, with I_0 being the amperage of the current source.

3.8 Source Superposition

When dealing with multiple sources, it can be difficult to analyze a circuit. In the circuit below, we need to find V_x flowing through the 4Ω resistor.



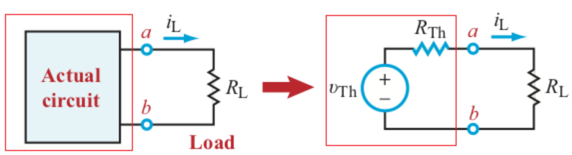
In this technique, we'd do the following

1. Remove each current/voltage source until only one remains.
   - Replace voltage sources with short circuits
   - Replace current sources with open circuits
2. Calculate voltage for the modified circuit
3. Repeat for all sources
4. Add voltages to get the final result

3.9 Thévenin Equivalent Circuits

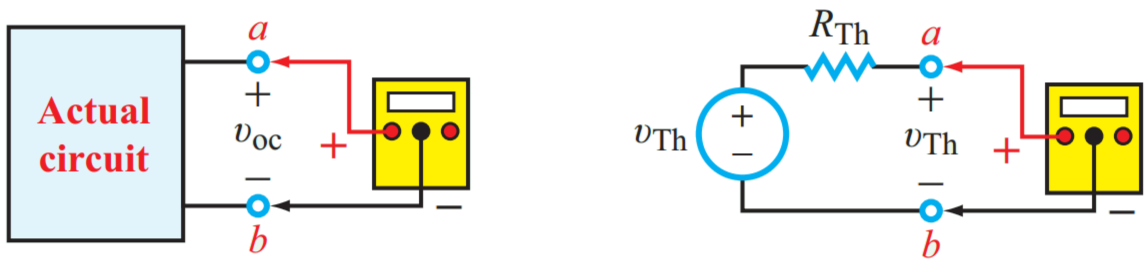
Thévenin was an old pompus french guy who got things named after him.

Given an input circuit, that affects a current circuit, it can be simplified to a basic voltage source and resistor. This voltage source and resistor is an imaginary circuit that does not exist. It merely mimics the behavior.



(a) Original circuit (b) Thévenin equivalent

As seen before, the black box can be simplified to a single voltage source called the Thévenin voltage and the resistor is the Thévenin resistor. The Thévenin voltage is the voltage measured across the two as seen below.

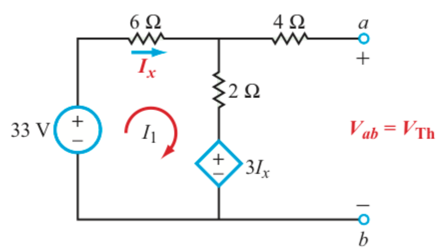


(a) Measuring v_{oc} on actual circuit (b) Measuring v_{Th} of equivalent circuit

In the first image, you can see there is a load resistor labelled R_L which we'd then be able to find the current (i_L) across using the Thévenin circuit. Now, to find the voltage and resistance.

To find the Thévenin voltage, you would turn the load resistor into an open circuit, then calculate the voltage drop across those two points using your voltage analysis method of choice.

As seen in the circuit below, the voltage across terminals a and b (V_{ab}) is equal to V_{th}. When doing nodal analysis, it's important to note below that the 4Ω resistor will NOT be used in analysis. Due to the open circuit, no current flows through that resistor.



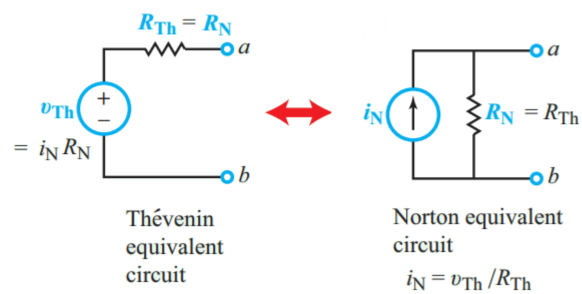
To find the Thévenin resistance, the way I like the most is to find the equivalent resistance of the complex circuit. To do this, we have to deactivate all the sources. We'd turn all the voltage sources into short circuits, and current sources into open circuits.

3.10 Norton Equivalent Circuits

A Norton equivalent circuit it like the current equivalent of the Thévenin circuit. The resistor has the same value, but now the voltage source is a current source where:

$$i_N = \frac{v_{th}}{R_{th}}$$

The relationship is seen in the below image.

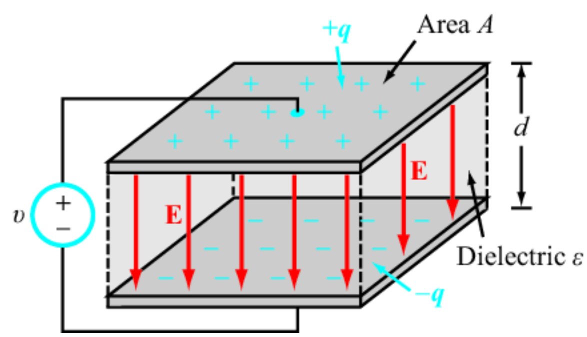


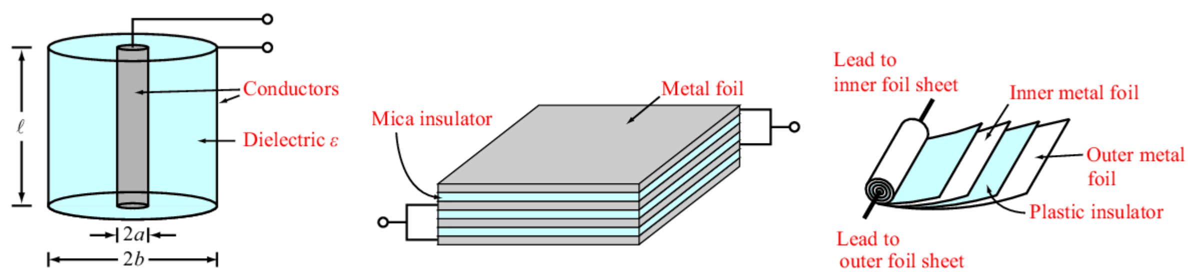
In the case of the Thévenin circuit, the resistor is in series, where in Norton circuits, the resistor is in parallel. Both circuits aren't real and are imaginary, used to model circuit behavior. The voltage and current splitting equations are NOT used here.

4 Circuit Components

4.1 Capacitors

Capacitors hold charge. A parallel plate capacitor is made of two identical metal plates with an insulator between them.





(a) Coaxial capacitor (b) Mica capacitor (c) Plastic foil capacitor

The insulator has an electric permittivity. The relative permittivity of a material is the following where $\epsilon_0 = 8.85 * 10^{-12}$ farads per meter (f/m)

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

An electric field E is induced in the insulator. The electrical susceptibility χ_e is how susceptible that material is to polarization. We get the following relation. Since free space contains no atoms, it's $\chi_e = 0$ and $\epsilon_r = 1$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

The electric field E formed by a parallel plate capacitor, where each plate has area A and charge q is given by.

$$E = \frac{q}{\epsilon A}$$

Then relating to voltage:

$$E = \frac{v}{d}$$

For any capacitor, regardless of type, has the capacitance C (measured in farads) as described by:

$$C = \frac{q}{v}$$

Given what we know, we can derive the capacitance for a parallel plate capacitor doing the following:

$$\begin{aligned}
E &= \frac{q}{\epsilon A} \\
E &= \frac{v}{d} \\
\frac{v}{d} &= \frac{q}{\epsilon A} \\
q &= \frac{\epsilon A v}{d} \\
C &= \frac{q}{v} \\
C &= \frac{\epsilon A v}{d v} \\
C &= \frac{\epsilon A}{d}
\end{aligned}$$

Then we can find the capacitance for a coaxial capacitor as the following where ℓ is the length, and b and a are the radii of the cylinders.

$$C = \frac{2\pi\epsilon\ell}{\ln(b/a)}$$

Now bringing in current, we see the following derivation

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

Then writing $v(t)$ in terms of $i(t)$:

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i d\tau$$

The derivation for instantaneous power of a capacitor:

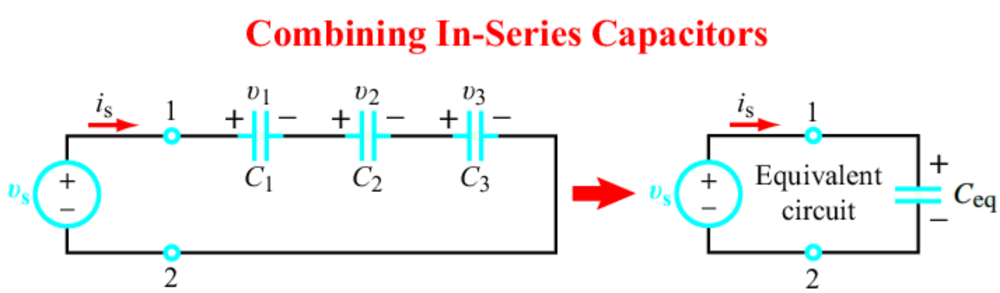
$$p(t) = vi$$
$$p(t) = Cv\frac{dv}{dt}$$

The derivation for instantaneous energy of a capacitor:

$$w(t) = \int_{\infty}^{t} p d\tau$$
$$w(t) = C\int_{\infty}^{t} (v\frac{dv}{d\tau}) d\tau$$
$$w(t) = C\int_{\infty}^{t} (\frac{1}{2}v^2\frac{d}{d\tau}) d\tau$$
$$w(t) = \frac{1}{2}Cv^2$$

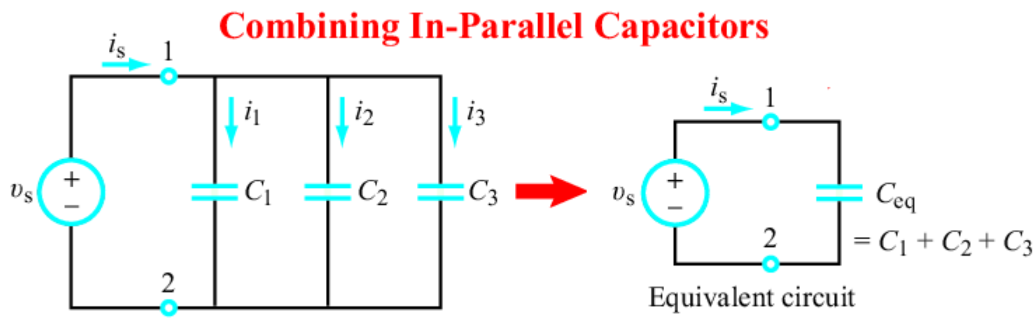
4.1.1 In Series

Capacitors in series get equivalent capacitance the way resistors do in parallel.



4.1.2 In Parallel

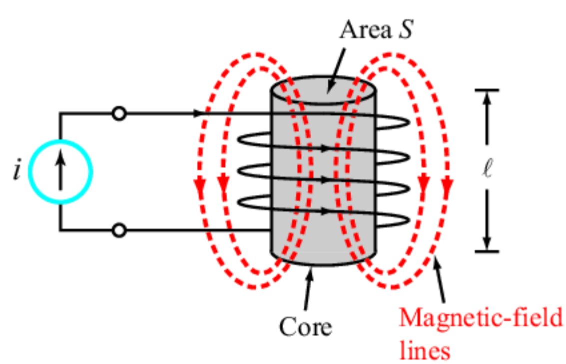
Capacitors in parallel get equivalent capacitance the way resistors do in series.



In both series and parallel, capacitors act as an open circuit since there is no current flowing through it in a stable state.

4.2 Inductors

Inductor : A current carrying conductor
Solenoid : Multiple turns of wire wound around an cylindrical core
Self Inductance : The magnetic flux linkage of a coil with itself
Mutual Inductance : The magnetic flux linkage of a coil due to the magnetic field generated by another coil
Inductance : Typically refers to self inductance



The inductance of a solenoid with N turns and magnetic permeability μ is:

$$L = \frac{\mu N^2 S}{\ell}$$

The magnetic flux linkage is:

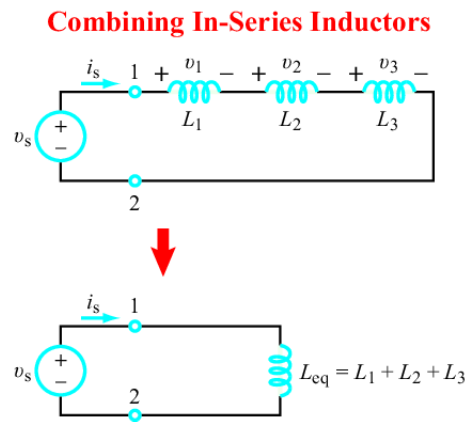
$$\Lambda = i\frac{\mu N^2 S}{\ell}$$

The inductance for any conducting system is:

$$L = \frac{\Lambda}{i}$$

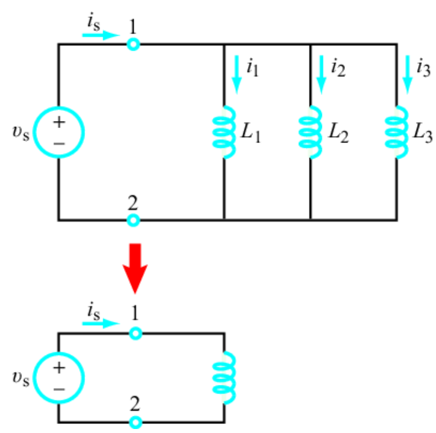
4.2.1 In Series

Inductors in series get equivalent inductance the way resistors do in series.



4.2.2 In Parallel

Inductors in parallel get equivalent inductance the way resistors do in parallel.



Both in series and in parallel, inductors act as short circuts, since they're just a coil of wire.

5 RLC Circuits

RC Circuits : A circuit composed of resistors and capacitors
RL Circuits : A circuit composed of resistors and inductors
RLC Circuits : A circuit composed of resistors, capacitors, and inductors
First Order Circuit : A circuit with only one energy storage unit **Natural Response :** The variations in voltage and current over time as a capacitor discharges